Most of the required items are worth ½ point and are highlighted in red

The only items worth more than ½ point are highlighted in green

Pink and blue highlights are used to distinguish graphs and do not correspond to any points

[2][a] OVER POLAR AXIS

$(r, -\theta)$: $r = 2\cos 2(-\theta)$ $\qquad$ $r = 2 + \cos 2(-\theta)$

$\frac{1}{2}$ $\boxed{\begin{array}{l} r = 2\cos(-2\theta) \\ r = 2\cos 2\theta \end{array}}$ $\qquad$ $\boxed{\begin{array}{l} r = 2 + \cos(-2\theta) \\ r = 2 + \cos 2\theta \end{array}}$ $\frac{1}{2}$

BOTH GRAPHS SYM OVER POLAR AXIS

OVER POLE

$(r, \pi + \theta)$: $r = 2\cos 2(\pi + \theta)$ $\qquad$ $r = 2 + \cos 2(\pi + \theta)$

PERIOD OF $\cos x$ IS 2π

$\frac{1}{2}$ $\boxed{\begin{array}{l} r = 2\cos(2\pi + 2\theta) \\ r = 2\cos 2\theta \end{array}}$ $\qquad$ $\boxed{\begin{array}{l} r = 2 + \cos(2\pi + 2\theta) \\ r = 2 + \cos 2\theta \end{array}}$ $\frac{1}{2}$

BOTH GRAPHS SYM OVER POLE

$\frac{1}{2}$

SO, BOTH GRAPHS $\boxed{\text{AUTOMATICALLY SYM OVER } \theta = \frac{\pi}{2}}$ ALSO

[b] ① $\theta = 2 + \cos 2\theta$

$\frac{1}{2}$ $\cos 2\theta = -2 \notin [-1, 1]$

NO θ FOR WHICH $r = 2 + \cos 2\theta$ PASSES THROUGH POLE

SO, NO INTERSECTION AT POLE

② $2\cos 2\theta = 2 + \cos 2\theta$, $\theta \in [0, 2\pi]$

$\frac{1}{2}$ $\cos 2\theta = 2 \notin [-1, 1]$

NO INTERSECTION WITH SAME θ ON BOTH CURVES

③ $-r = 2\cos 2(\pi + \theta)$ $\frac{1}{2}$

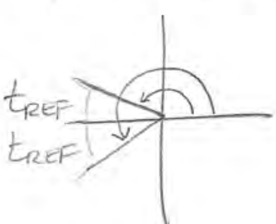
$-r = 2\cos 2\theta$

$r = -2\cos 2\theta$

$t_{REF} = \cos^{-1}\frac{2}{3}$

$-2\cos 2\theta = 2 + \cos 2\theta$ $\frac{1}{2}$, $\theta \in [0, \pi]$

$\cos 2\theta = -\frac{2}{3}$ $\frac{1}{2}$



$\cos t = -\frac{2}{3}$ $\quad t = 2\theta \in [0, 2\pi]$

$2\theta = t = \pi - \cos^{-1}\frac{2}{3}$ OR $\pi + \cos^{-1}\frac{2}{3}$

$\boxed{\theta = \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3} \text{ OR } \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}}$ 1

$\theta = \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}$ ON $r = 2 + \cos 2\theta$

$r = 2 + \cos 2\left(\frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)$

$\left(\frac{1}{2}\right) = \boxed{2 + \cos\left(\pi - \cos^{-1}\frac{2}{3}\right)}$

ALSO ACCEPTABLE $2 - \cos\left(\cos^{-1}\frac{2}{3}\right) \longrightarrow$ (1) $= \boxed{2 + \cos\pi\cos\left(\cos^{-1}\frac{2}{3}\right) + \cancel{\sin\pi}^{\,0}\sin\left(\cos^{-1}\frac{2}{3}\right)}$

$= 2 + -\frac{2}{3} = \boxed{\frac{4}{3}}$ $\left(\frac{1}{2}\right)$

$\theta = \pi + \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}$ ON $r = 2\cos 2\theta$

$= \frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}$

$r = 2\cos 2\left(\frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)$

$\left(\frac{1}{2}\right) = \boxed{2\cos\left(3\pi - \cos^{-1}\frac{2}{3}\right)}$

$\left(\frac{1}{2}\right) = \boxed{2\cos\left(\pi - \cos^{-1}\frac{2}{3}\right)}$

ABOVE $\rightarrow = 2\left(-\frac{2}{3}\right) = \boxed{-\frac{4}{3}}$ $\left(\frac{1}{2}\right)$

CURVES INTERSECT IN Q_1 AT

$\left(\frac{1}{2}\right)$ $\boxed{\begin{array}{l}\left(-\frac{4}{3}, \frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right) \\ \left(\frac{4}{3}, \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)\end{array}}$ $\begin{array}{l}\text{ON } r = 2\cos 2\theta \\ \text{ON } r = 2 + \cos 2\theta\end{array}$

$\theta = \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}$ ON $r = 2 + \cos 2\theta$

$r = \boxed{2 + \cos\left(\pi + \cos^{-1}\frac{2}{3}\right)}$ $\left(\frac{1}{2}\right)$

ALSO ACCEPTABLE $2 - \cos\left(\cos^{-1}\frac{2}{3}\right) \longrightarrow$ (1) $= \boxed{2 + \cos\pi\cos\left(\cos^{-1}\frac{2}{3}\right) - \cancel{\sin\pi}^{\,0}\sin\left(\cos^{-1}\frac{2}{3}\right)}$

$= 2 + -\frac{2}{3} = \boxed{\frac{4}{3}}$ $\left(\frac{1}{2}\right)$

$\theta = \pi + \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}$ ON $r = 2\cos 2\theta$

$= \frac{3\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}$

$r = \boxed{2\cos\left(3\pi + \cos^{-1}\frac{2}{3}\right)}$ $\left(\frac{1}{2}\right)$

$= \boxed{2\cos\left(\pi + \cos^{-1}\frac{2}{3}\right)}$ $\left(\frac{1}{2}\right)$

ABOVE $\rightarrow = 2\left(-\frac{2}{3}\right) = \boxed{-\frac{4}{3}}$ $\left(\frac{1}{2}\right)$

CURVES INTERSECT IN Q_2 AT

$\left(\frac{1}{2}\right)$ $\boxed{\begin{array}{l}\left(-\frac{4}{3}, \frac{3\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}\right) \\ \left(\frac{4}{3}, \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}\right)\end{array}}$ $\begin{array}{l}\text{ON } r = 2\cos 2\theta \\ \text{ON } r = 2 + \cos 2\theta\end{array}$

4 $\boxed{-r = 2 + \cos 2(\pi + \theta)}$ $\frac{1}{2}$

$$-r = 2 + \cos 2\theta$$

$$r = -2 - \cos 2\theta$$

$\boxed{2\cos 2\theta = -2 - \cos 2\theta}$ $\frac{1}{2}$

$\boxed{\cos 2\theta = -\frac{2}{3}}$ $\frac{1}{2}$

FROM 3 $\rightarrow$ $\boxed{\theta = \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3} \text{ OR } \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}}$ $\frac{1}{2}$

$\theta = \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}$ ON $r = 2\cos 2\theta$

$r = \boxed{2\cos\left(\pi - \cos^{-1}\frac{2}{3}\right)}$ $\frac{1}{2}$

FROM 3 $\rightarrow$ $= \boxed{-\frac{4}{3}}$ $\frac{1}{2}$

$\theta = \pi + \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}$ ON $r = 2 + \cos 2\theta$

$= \frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}$ $\quad r = \boxed{2 + \cos\left(3\pi - \cos^{-1}\frac{2}{3}\right)}$ $\frac{1}{2}$

FROM 3 $\longrightarrow$ $= \boxed{\frac{4}{3}}$ $\frac{1}{2}$

CURVES INTERSECT IN Q_3 AT

$\frac{1}{2}$ $\left(-\frac{4}{3}, \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)$ ON $r = 2\cos 2\theta$

$\left(\frac{4}{3}, \frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)$ ON $r = 2 + \cos 2\theta$

$\theta = \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}$ ON $r = 2\cos 2\theta$

$r = \boxed{2\cos\left(\pi + \cos^{-1}\frac{2}{3}\right)}$ $\frac{1}{2}$

FROM 3 $\rightarrow$ $= \boxed{-\frac{4}{3}}$ $\frac{1}{2}$

$\theta = \pi + \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}$ ON $r = 2 + \cos 2\theta$

$= \frac{3\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}$ $\quad r = \boxed{2 + \cos\left(3\pi + \cos^{-1}\frac{2}{3}\right)}$ $\frac{1}{2}$

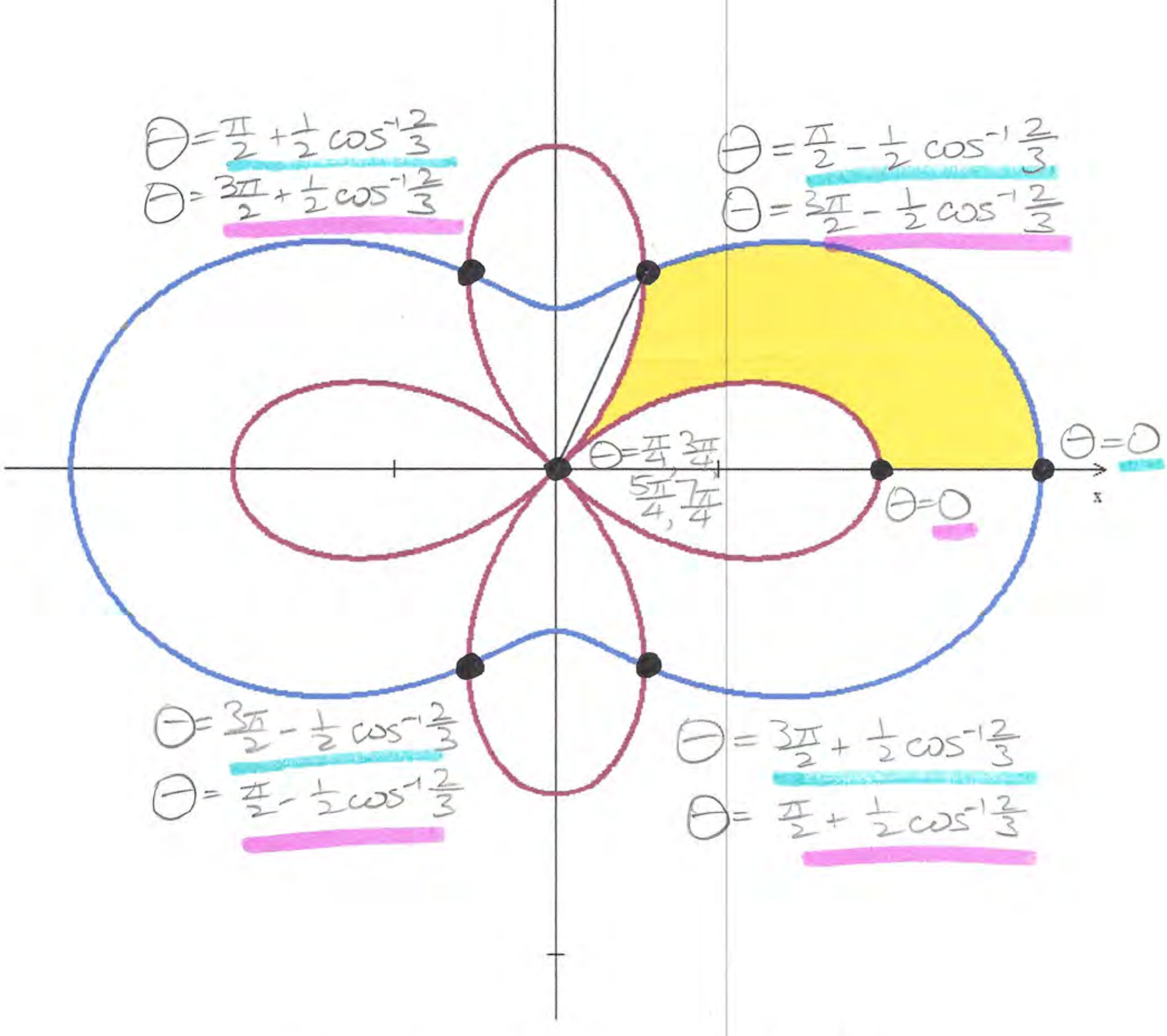
FROM 3 $\rightarrow$ $= \boxed{\frac{4}{3}}$ $\frac{1}{2}$

CURVES INTERSECT IN Q_4 AT

$\frac{1}{2}$ $\left(-\frac{4}{3}, \frac{\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}\right)$ ON $r = 2\cos 2\theta$

$\left(\frac{4}{3}, \frac{3\pi}{2} + \frac{1}{2}\cos^{-1}\frac{2}{3}\right)$ ON $r = 2 + \cos 2\theta$

NO POINTS ON THIS PAGE



[c] $(r,\theta) = \left(-\frac{4}{3}, \frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)$ $\frac{1}{2}$

$(x,y) = \left(\boxed{-\frac{4}{3}\cos\left(\frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)}, -\frac{4}{3}\sin\left(\frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right)\right)$

$= \left(-\frac{4}{3}\cdot -\frac{1}{\sqrt{6}}, -\frac{4}{3}\cdot -\sqrt{\frac{5}{6}}\right) = \left(\frac{4}{3\sqrt{6}}, \frac{4\sqrt{5}}{3\sqrt{6}}\right)$

$\cos\left(\frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right) = \boxed{\cos\frac{3\pi}{2}\cos\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right) + \sin\frac{3\pi}{2}\sin\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right)}$ 1 $\frac{1}{2}$

ALSO ACCEPTABLE $\longrightarrow$

$-\cos\left(\frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right) = -\sin\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right)$

MUST HAVE BOTH EXPRESSIONS

$= -\sin\left(\underbrace{\frac{1}{2}\cos^{-1}\frac{2}{3}}_{\alpha}\right) = \boxed{-\sqrt{\frac{1-\frac{2}{3}}{2}}} = \boxed{-\frac{1}{\sqrt{6}}}$ $\frac{1}{2}$

$\cos\alpha = \frac{2}{3}$ AND α IS ACUTE

$\sin\left(\frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right) = \boxed{\sin\frac{3\pi}{2}\cos\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right) - \cos\frac{3\pi}{2}\sin\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right)}$ 1 $\frac{1}{2}$

ALSO ACCEPTABLE $\longrightarrow$

$-\sin\left(\frac{\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}\right) = -\cos\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right)$

MUST HAVE BOTH EXPRESSIONS

$= -\cos\left(\frac{1}{2}\cos^{-1}\frac{2}{3}\right) = \boxed{-\sqrt{\frac{1+\frac{2}{3}}{2}}} = \boxed{-\frac{\sqrt{5}}{\sqrt{6}}}$ $\frac{1}{2}$

$r = 2\cos 2\theta \rightarrow x = 2\cos 2\theta\cos\theta$

$y = 2\cos 2\theta\sin\theta$

$\frac{dy}{dx} = \frac{\boxed{\cancel{2}(-2\sin 2\theta\sin\theta + \cos 2\theta\cos\theta)}}{\boxed{\cancel{2}(-2\sin 2\theta\cos\theta - \cos 2\theta\sin\theta)}}$ 1 1

AT $\theta = \frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}$

$\cos\theta = -\frac{1}{\sqrt{6}}$, $\sin\theta = -\sqrt{\frac{5}{6}}$ FROM ABOVE

$\cos 2\theta = -\frac{2}{3}$ FROM [b] 3

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(-\frac{1}{\sqrt{6}}\right)\left(-\sqrt{\frac{5}{6}}\right) = \frac{2\sqrt{5}}{6}$

SO $\left.\frac{dy}{dx}\right|_{\theta = \frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3}} \rightarrow \boxed{\frac{-\cancel{2}\left(\frac{2\sqrt{5}}{\cancel{6}3}\right)\left(-\frac{\sqrt{5}}{\sqrt{6}}\right) + \left(-\frac{2}{3}\right)\left(-\frac{1}{\sqrt{6}}\right)}{-\cancel{2}\left(\frac{2\sqrt{5}}{\cancel{6}3}\right)\left(-\frac{1}{\sqrt{6}}\right) - \left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{\sqrt{6}}\right)}} \frac{3\sqrt{6}}{3\sqrt{6}}$ 1

$\rightarrow \frac{10+2}{2\sqrt{5} - 2\sqrt{5}} \longrightarrow \frac{12}{0}$ $\leftarrow$ NON-ZERO, $\leftarrow$ ZERO $\frac{1}{2}$

UNDEFINED

V.T.L.

1 $\boxed{x = \frac{4}{3\sqrt{6}}} = \frac{2\sqrt{6}}{9}$

SUBTRACT $\frac{1}{2}$ POINT IF "x =" MISSING

[d] $\theta = 0 \longrightarrow r = 2+\cos 2(0) = 2+1 = 3 \longrightarrow (3,0)$

$\theta = 0 \longrightarrow r = 2\cos 2(0) = 2(1) = 2 \longrightarrow (2,0)$

$2\cos 2\theta = 0 \longrightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ $\longleftarrow$ LECTURE

JUST PAST $\theta = 0 \longrightarrow \theta = \frac{\pi}{4}$

JUST BEFORE $\theta = \frac{3\pi}{2} - \frac{1}{2}\cos^{-1}\frac{2}{3} \longrightarrow \theta = \frac{5\pi}{4}$

$$A = \underbrace{\boxed{\tfrac{1}{2}}}_{\frac{1}{2}} \left[\underbrace{\boxed{\int_0^{\frac{\pi}{2}-\frac{1}{2}\cos^{-1}\frac{2}{3}}}}_{1} \underbrace{\boxed{(2+\cos 2\theta)^2}}_{\frac{1}{2}} d\theta - \overset{1}{\boxed{\int_0^{\frac{\pi}{4}}}} \overset{\frac{1}{2}}{\boxed{(2\cos 2\theta)^2}} d\theta - \overset{1}{\boxed{\int_{\frac{5\pi}{4}}^{\frac{3\pi}{2}-\frac{1}{2}\cos^{-1}\frac{2}{3}}}} \overset{\frac{1}{2}}{\boxed{(2\cos 2\theta)^2}} d\theta \right]$$